Two-temperature generalized thermoelasticity under ramp-type heating by finite element method

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Abstract

In this work, a general finite element model is proposed to analyze transient phenomena in thermoelastic half-space filled with an elastic material, which has constant elastic parameters. The governing equations are taken in the context of the two-temperature generalized thermoelasticity theory (Youssef in IMA J. Appl. Math. 71(3):383–390, 2006). A linear temperature ramping function is used to more realistically model thermal loading of the half-space surface. The medium is assumed initially quiescent. A finite element scheme is presented for the high accuracy numerical purpose. The numerical solutions of the non-dimensional governing partial differential equations of the problem have been shown graphically and some comparisons have been shown in figures to estimate the effect of the ramping parameter of heating and the parameter of two-temperature.

Keywords

Thermoelasticity Two-temperature thermoelasticity Ramp-type heating Finite element

1 Introduction

The generalized thermoelasticity theory is receiving serious attention of different researchers. Because of the advancement of pulsed lasers, fast burst nuclear reactors and particle accelerators, etc. which can supply heat pulses with a very fast time-rise, Bargmann [1], Anisimov et al. [2], Boley [3], Qiu and Tien [4], Tzou [5], Chen et al. [6]. Chandrasekharaiah [7] has developed the second sound effect. Now, mainly two different models of generalized thermoelasticity are being extensively used: one proposed by Lord and Shulman [8] and the other proposed by Green and Lindsay [9]. The L-S theory suggests one relaxation time and, according to this theory, only Fourier's heat conduction equation is modified; while G-L theory suggests two relaxation times, and both the energy equation and the equation of motion are modified. Erbay and Suhubi [10] studied longitudinal wave propagation in an infinite circular cylinder, which is assumed to be made of the generalized thermoelastic material, and thereby obtained the dispersion relation when the surface temperature of the cylinder was kept constant.

Chen and Gurtin [11], Chen et al. [12] have formulated a theory of heat conduction in deformable bodies, which depends upon two distinct temperatures, the conductive temperature ϕ and the thermo-dynamic temperature T. For time independent situations, the difference between these two temperatures is proportional to the heat supply, and in the absence of any heat supply, the two temperatures are identical Chen et al. [13]. For time dependent problems, however, and for wave propagation problems in particular, the two temperatures are in general different regardless of the presence of a heat supply. The two temperatures T,φ and the strain are found to have representations in the form of a traveling wave plus a response, which occurs instantaneously throughout the body Boley [14].

Warren and Chen [15] studied the wave propagation in the two-temperature theory of thermoelasticity, while Youssef [16] investigated this theory in the context of the generalized theory of thermoelasticity. Youssef and Abbas [17], Abbas and Abd-alla [18], Abbas and Youssef [19], Abbas and Othman [20, 21] and Abbas [22, 23], applied the finite element method in different generalized thermoelastic problems.

The exact solution of a nonlinear model of the thermal shock problem of a generalized thermoelastic half-space of two-temperature theory exists only for very special and simple initial- and boundary problem. In view of calculating general problems, a numerical solution technique is to be used; for this reason, the finite element method is chosen. The Finite element method is a powerful technique originally developed for numerical solution of complex problems in structural mechanics, and it remains the method of choice for complex systems. A further benefit of this method is that it allows physical effects to be visualized and quantified regardless of experimental limitations.

The present investigation is devoted to a study of the induced temperature and stress fields in an elastic half space under the purview of two-temperature generalized thermoelasticity theory. The half space continuum is considered to be made of an isotropic homogeneous thermoelastic material, the bounding plane surface being subjected to a ramp-type heating. The rationale behind the study of such a type of heating is that, the temperature of the bounding surface cannot be elevated instantaneously, since a finite rise time in temperature is required for this purpose.

2 Basic equations and formulation

In the context of two-temperature generalized thermoelasticity (TTGTE), the linear equations governing thermoelastic interactions in a homogeneous isotropic medium are in the forms [16]:

The equations of motion are in the form

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ji} + F_i - \gamma T_{,i} = \rho \ddot{u}_i.$$

(1)

The heat equation is in the form

$$K\phi_{,ii} = \left(\frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2}\right)[\rho C_e T + T_o \gamma e_{ii}],$$

(2)
and

$$\phi - T = a\phi_{,ii}.$$

(3)
The constitutive stress equations are in the form

$$\sigma_{ij} = \mu(u_{i,j} + u_{j,i}) + [\lambda u_{i,i} - \gamma T]\delta_{ij},$$

(4)
where a>0 is constant and is called the two-temperature parameter as when a=0 gives the case of (GTE), λ, μ is the Lame's constants, ρ is the mass density, K is the thermal conductivity, c_e is the specific heat at constant strain, τ_0 is the relaxation time, T is the dynamical temperature above reference temperature T_o, φ is the conductive temperature, $\gamma=\alpha_T(3\lambda+2\mu)$ is the stress-temperature modulus, in which α_T is the coefficient of linear thermal expansion, t is the time, σ_{ij} is the components of stress tensor, e_{ij} is the components of strain tensor, u_i is the components of displacement vector and F_i is the body force vector.

3 Formulation of the problem

We consider a half-space (x≥0) with the x-axis pointing into the medium. This half-space is subjected on the bounding plane (x=0) to thermal and/or mechanical effects that depend on the time t and the spatial coordinate y ($-\infty<y<\infty$). We assume that there are no body forces or heat sources affecting the medium and it is initially quiets.

The displacement vector has the components:

$$u = u(x, y, t), \qquad v = v(x, y, t),$$

$$w(x, y, t) = 0,$$

(5)
where

$$u(x, y, t)|_{x\to\infty} = u(x, y, t)|_{y\to\infty} = v(x, y, t)|_{x\to\infty}$$
$$= v(x, y, t)|_{y\to\infty} = 0.$$

From Eq. (2), we have

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = \left[\frac{\partial}{\partial t} + \tau_o \frac{\partial^2}{\partial t^2}\right]\left[\frac{\rho c_e}{K}T + \frac{\gamma T_o}{K}e\right],$$

(6)
and

$$\phi - T = a\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right),$$

(7)
where

$$\phi(x,y,t)|_{x\to\infty} = T(x,y,t)|_{x\to\infty} = \phi(x,y,t)|_{y\to\infty} = T(x,y,t)|_{y\to\infty} = 0.$$

From Eq. (1), we obtain

$$\rho\ddot{u} = (\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 u}{\partial y^2} + (\lambda+\mu)\frac{\partial^2 v}{\partial x\partial y} - \gamma\frac{\partial T}{\partial x},$$

(8)

$$\rho\ddot{v} = (\lambda+2\mu)\frac{\partial^2 v}{\partial y^2} + \mu\frac{\partial^2 v}{\partial x^2} + (\lambda+\mu)\frac{\partial^2 u}{\partial x\partial y} - \gamma\frac{\partial T}{\partial y}.$$

(9)

From Eq. (4), the constitutive equations can be written as

$$\sigma_{xx} = (\lambda+2\mu)e - 2\mu\frac{\partial v}{\partial y} - \gamma T,$$

(10)

$$\sigma_{yy} = (\lambda+2\mu)e - 2\mu\frac{\partial u}{\partial x} - \gamma T,$$

(11)

$$\sigma_{zz} = \lambda e - \gamma T,$$

(12)

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),$$

(13)

$$\sigma_{xz} = \sigma_{yz} = 0,$$

(14)

where

$$e = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}.$$

(15)

For simplicity, we shall use the following non-dimensional variables:

$$x' = c_0\chi x, \quad y' = c_0\chi x, \quad \tau_0' = c_0^2\chi\tau_0,$$
$$t' = c_0^2\chi t, \quad u' = c_0\chi u, \quad v' = c_0\chi v,$$
$$\sigma_{ij}' = \frac{\sigma_{ij}}{\mu}, \quad \theta' = \frac{\gamma(T-T_o)}{(\lambda+2\mu)},$$
$$\phi' = \frac{\gamma(\phi-T_o)}{(\lambda+2\mu)},$$

(16)

where $c_0 = \sqrt{\frac{\lambda+2\mu}{\rho}}$ is the longitudinal wave speed and $\chi = \frac{\rho c_e}{K}$ is the thermal viscosity.

Using these non-dimensional variables, Eqs. (6)–(15) take the form

$$\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2} = \left[\frac{\partial}{\partial t}+\tau_o\frac{\partial^2}{\partial t^2}\right]\left[\theta+\varepsilon\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right],$$

(17)

$$\phi-\theta=\Omega\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}\right),$$

(18)

where $\Omega = ac_0^2\chi^2$ is the dimensionless two-temperature parameter and $\varepsilon = \frac{\gamma^2 T_o}{\rho c_e(\lambda+2\mu)}$ is the dimensionless thermoelastic coupling constant

$$\beta^2\ddot{u}=\beta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+(\beta^2-1)\frac{\partial^2 v}{\partial x\partial y}-\beta^2\frac{\partial\theta}{\partial x},$$

(19)

$$\beta^2\ddot{v}=\beta^2\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial x^2}+(\beta^2-1)\frac{\partial^2 u}{\partial x\partial y}-\beta^2\frac{\partial\theta}{\partial y},$$

(20)

$$\sigma_{xx}=\beta^2 e-2\frac{\partial v}{\partial y}-\beta^2\theta,$$

(21)

$$\sigma_{yy}=\beta^2 e-2\frac{\partial u}{\partial x}-\beta^2\theta,$$

(22)

$$\sigma_{zz}=\left(\beta^2-2\right)e-\beta^2\theta,$$

(23)

$$\sigma_{xy}=\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right),$$

(24)

$$e=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y},$$

(25)

where $\beta^2 = \frac{\lambda+2\mu}{\mu}$ is the ratio of the longitudinal waves speed to the shear waves speed.

4 The boundary conditions

We consider the problem of a half-space Ψ, which is defined as follows

$$\Psi=\{(x,y,z):0\le x<\infty,\ -\infty<y<\infty,\ -\infty<z<\infty\}.$$

The boundary of the half-space (x=0) is affected by ramp-type heating and harmonically varying heat which depends on the time t of the form as following

$$\phi(0,y,t)=\begin{cases}0 & t\le 0,\\ \phi_1\frac{t}{t_o} & 0<t\le t_o,\\ \phi_1 & t>t_o,\end{cases}$$

(26)

where t_o indicates the length of time to rise the heat and ϕ_1 is constant, this means that the boundary of the half-space, which is initially at rest and has a fixed temperature T_o, is suddenly raised to an increasing temperature equal to the function $G(t) = \phi_1 \frac{t}{t_o}$ and after the instance $t=t_o$ coming we let the temperature with constant value ϕ_1 maintained from then on. We assume that, on the boundary $x=0$ the displacement u of the body does not depends on x, hence we have

$$u'(0, y, t) = 0, \tag{27}$$

and the medium is subjected to a rough and rigid foundation enough to prevent the displacement v at any time and any point of y, then, we have

$$v(0, y, t) = 0. \tag{28}$$

5 Finite element method

In order to investigate two-dimensional problem of two-temperature generalized thermoelastic half-space subjected to ramp-type heating by finite element method (FEM). The numerical solutions of the non-dimensional governing equations (17)–(20) using the finite element method (FEM), the weak formulations of these equations are derived. It is convenient to prescribe the set of independent test functions to consist of the displacement components u, v and the temperatures θ, ϕ. To obtain the weak formulation, the governing equations are multiplied by independent weighting functions and then are integrated over the spatial domain with the boundary. Applying integration by parts and making use of the divergence theorem reduces the order of the spatial derivatives and allows for the application of the boundary conditions. Using the well known Galerkin procedure, the unknown fields u, v, θ and ϕ, and the corresponding weighting functions are approximated by the same shape functions, which are defined piecewise on the elements. The last step towards the finite element discretization is to choose the element type and the associated shape functions. Eight nodes of quadrilateral elements are used. The unknown fields are approximated either by linear shape functions, which are defined by four corner nodes or by quadratic shape functions, which are defined by all of the eight nodes (two-dimensional quadrilateral elements). On other hand the unknown fields are approximated either by linear shape functions, which are defined by three corner nodes or by quadratic shape functions, which are defined by all of the six nodes (two-dimensional triangular elements). The shape function is usually denoted by the letter N and is usually the coefficient that appears in the interpolation polynomial. A shape function is written for each individual node of a finite element and has the property that its magnitude is 1 at that node and 0 for all other nodes in that element. We assume that the master element has its local coordinates in the range [−1,1]. In our case, the two-dimensional quadrilateral elements are used, which is given by

1. (i)

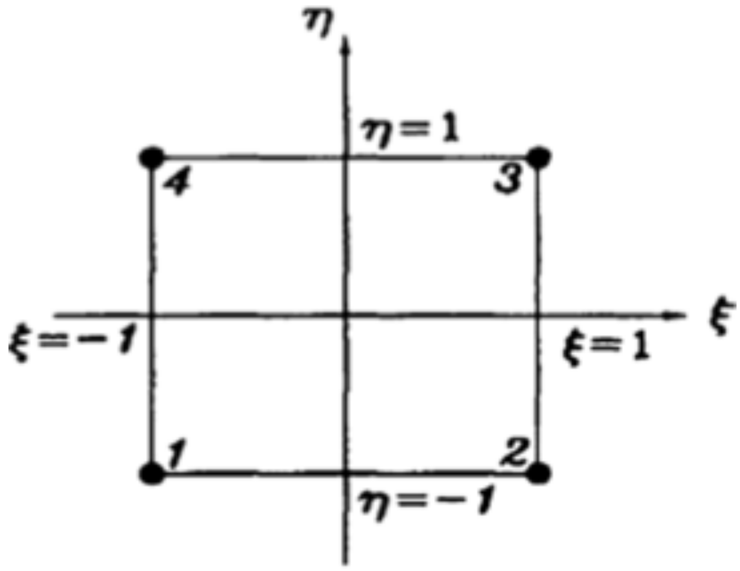
   Linear shape functions

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta),$$

$$N_2 = \frac{1}{4}(1+\xi)(1-\eta),$$

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta),$$

$$N_4 = \frac{1}{4}(1-\xi)(1+\eta),$$



(i) Sketch of linear shape functions

2. (ii)

Quadratic shape functions

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta)(-1-\xi-\eta),$$

$$N_5 = \frac{1}{2}(1-\xi^2)(1-\eta),$$

$$N_2 = \frac{1}{4}(1+\xi)(1-\eta)(-1+\xi-\eta),$$

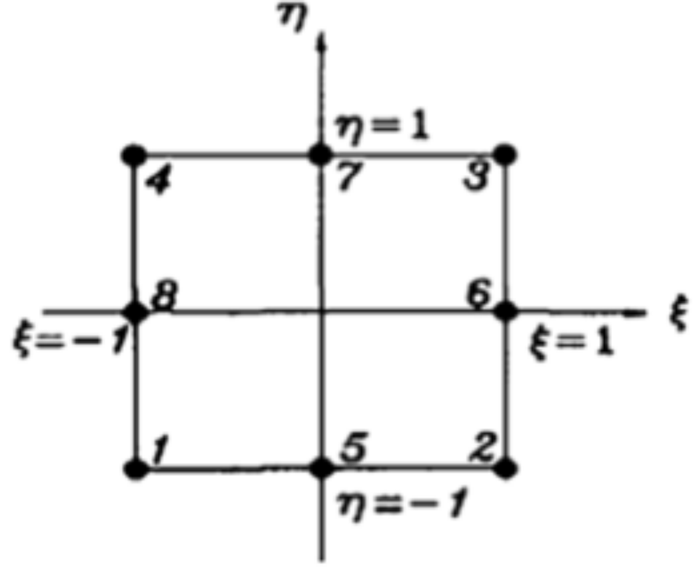
$$N_6 = \frac{1}{2}(1+\xi)(1-\eta^2),$$

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta)(-1+\xi+\eta),$$

$$N_7 = \frac{1}{2}(1-\xi^2)(1+\eta),$$

$$N_4 = \frac{1}{4}(1-\xi)(1+\eta)(-1-\xi+\eta),$$

$$N_8 = \frac{1}{2}(1-\xi)(1-\eta^2).$$



(ii) Sketch of quadratic shape functions

On the other hand, the time derivatives of the unknown variables have to be determined by Newmark time integration method (see Wriggers [24]).

6 Numerical results and discussion

The copper material was chosen for purposes of numerical evaluations and the constants of the problem were taken as following [19]:

$$\mathrm{K} = 386\ \mathrm{N/K\,s}, \qquad \alpha_T = 1.78(10)^{-5}\ \mathrm{K}^{-1},$$
$$C_e = 383.1\ \mathrm{m^2/K}, \qquad \eta = 8886.73\ \mathrm{m/s^2}$$
$$\mu = 3.86(10)^{10}\ \mathrm{N/m^2}, \qquad \lambda = 7.76(10)^{10}\ \mathrm{N/m^2},$$
$$\rho = 8954\ \mathrm{kg/m^3}, \qquad \tau_o = 0.02,$$
$$\mathrm{T_o} = 293\ \mathrm{K}, \qquad \varepsilon = 0.0168, \qquad \beta^2 = 4.$$

The computations were carried out for t=0.2, ϕ_1=1, and y=2.0. The conductive temperature, the dynamical temperature, the stress and the displacements distributions are represented graphically at different positions of x and different value of t_o.

Table 1 presents an analysis of grid effects. The grid size has been refined until the values of u, v, θ and ϕ, stabilizes. Further refinement of mesh size over 1000×50 elements does not change the values considerably. Thus, elements with x×y=1000×50 were used for this study.

Table 1

Grid independent test (t=0.2, t_o=0.1, τ_0=0.02, y=1, Ω=0.1, x=0.5)

Mesh size	u×10^{-3}	v×10^{-3}	θ×10^{-1}	ϕ×10^{-1}
100 × 50	3.049372	−7.953752	2.468374	3.566436
200 × 50	3.048257	−7.956228	2.460469	3.566472
300 × 50	3.048051	−7.956687	2.457867	3.566479
400 × 50	3.047978	−7.956847	2.456571	3.566481
500 × 50	3.047945	−7.956922	2.455796	3.566482
700 × 50	3.047916	−7.956986	2.454912	3.566483
900 × 50	3.047901	−7.957021	2.454250	3.566483
1000 × 50	3.047901	−7.957021	2.454250	3.566483

The field quantities, conductive temperature, dynamical temperature, stress and displacements depend not only on the state and space variables t, x and y, but also on the rise-time parameter t_0. It has been observed that, the finite rise-time parameter plays a vital role on the development of all the fields.

Here all the variables/parameters are taken in nondimensional forms. Numerical analysis has been carried out by taking x ranging from 0.0 to 1.5. The numerical values for the field quantities are computed for a wide range of values of finite pulse rise-time t_0 in the two situations $t<t_0$ and $t>t_0$. All figures have been drown for the two cases: Ω=0.0, which shows the case of the theory of (GTE) and has been described by solid line and Ω=0.1 which shows the case of the theory of (TTGTE) and has been described by dotted line in the following figures.

In Fig. 1, we display the conductive temperature for different values of t_0 to show its effect on the field in the two types (GTE) and (TTGTE) and we have noticed that:

1. 1.

   In the (TTGTE) theory the speed of the wave propagation of the conductive temperature vanished at larger distances than in the (GTE) theory.

2. 2.

The ramping parameter t_o has a significant effect on the conductive temperature for the two theories.

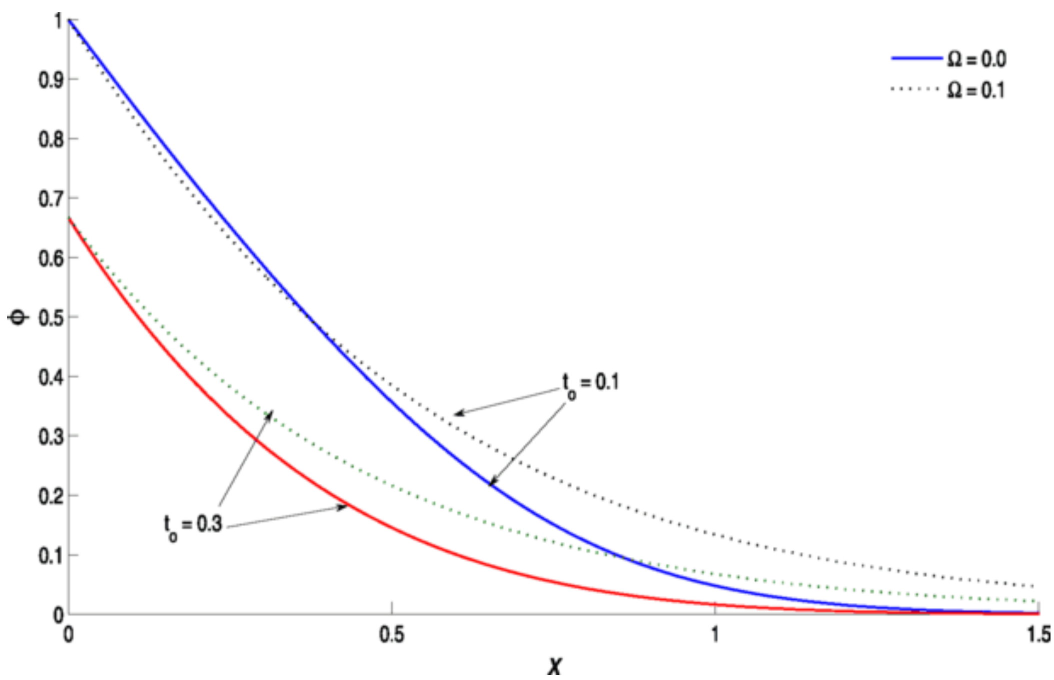


Fig. 1

The conductive temperature distribution ϕ for different values of t_o and Ω

In Fig. 2, we display the dynamical temperature for different values of t_o to show its effect on the filed in the two types (GTE) and (TTGTE) and we have noticed that:

1. 1.

   In the (TTGTE) theory the speed of the wave propagation of the dynamical temperature vanished at larger distant than in the (GTE) theory.

2. 2.

   The ramping parameter t_o has a significant effect on the conductive temperature for the two theories.

3. 3.

   The curves of the dynamical temperature coincide with the curves of the conductive temperature when Ω=0.0 in Fig. 1.

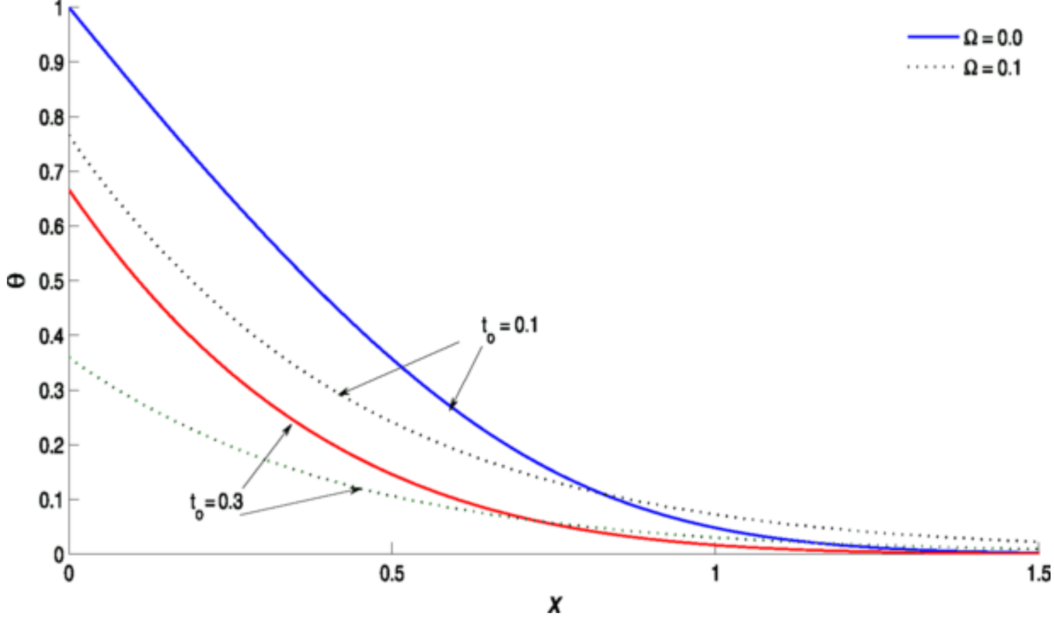


Fig. 2

The dynamical temperature distribution θ for different values of t_o and Ω

In Figs. 3 and 4, we display the displacements u and v for different values of t_o to show its effect on the filed in the two types (GTE) and (TTGTE) and we have noticed that:

1. 1.

   In the (TTGTE) theory the speed of the wave propagation of the displacements u and v vanished at larger distant than in the (GTE) theory.

2. 2.

   The ramping parameter t_o has a significant effect on the displacements u and v for the two theories.

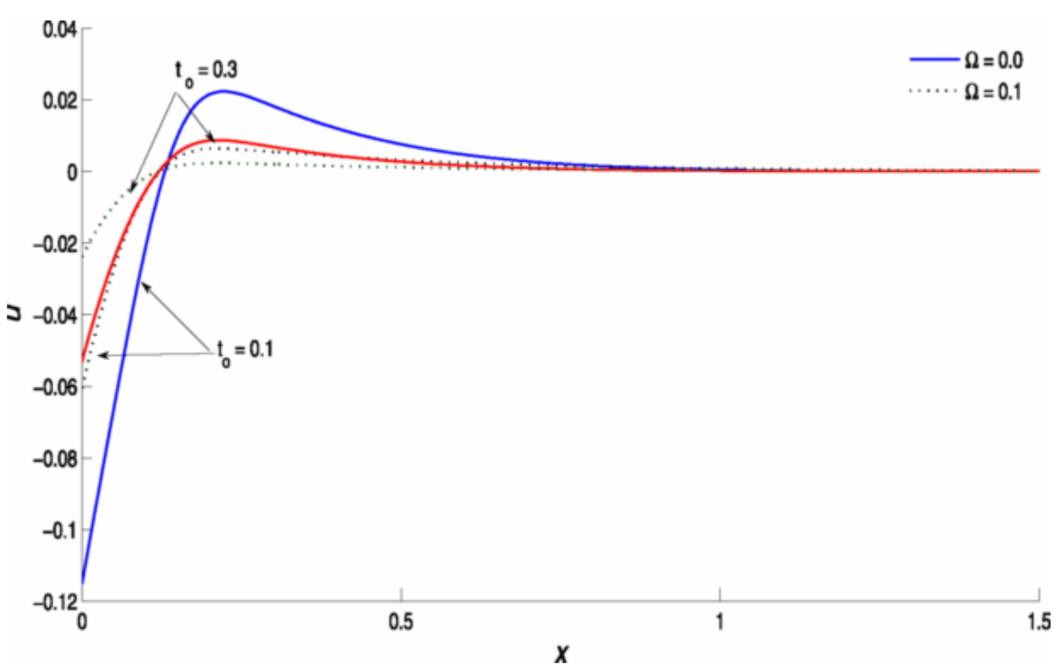


Fig. 3

The displacement distribution u for different values of t_o and Ω

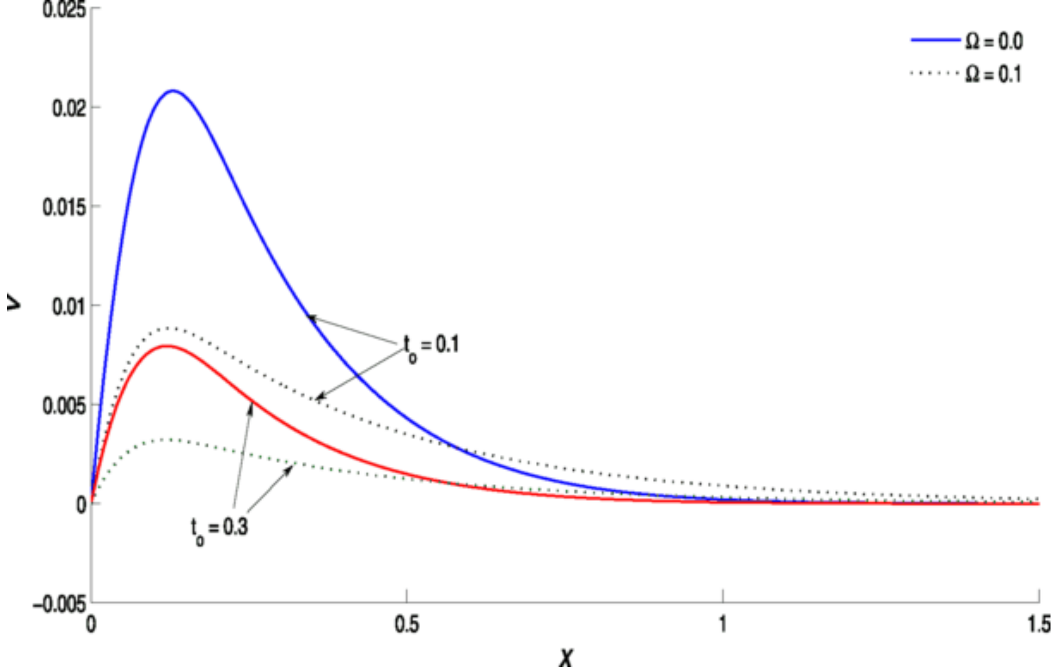


Fig. 4

The displacement distribution v for different values of t_o and Ω

In Figs. 5 and 6, we display the stresses σ_{xx} and σ_{yy} for different values of t_o to show its effect on the filed in the two types (GTE) and (TTGTE) and we have noticed that:

1. 1.

   In the (TTGTE) theory the speed of the wave propagation of the stresses σ_{xx} and σ_{yy} vanished at larger distant than in the (GTE) theory.

2. 2.

   The ramping parameter t_o has a significant effect on the stresses σ_{xx} and σ_{yy} for the two theories.

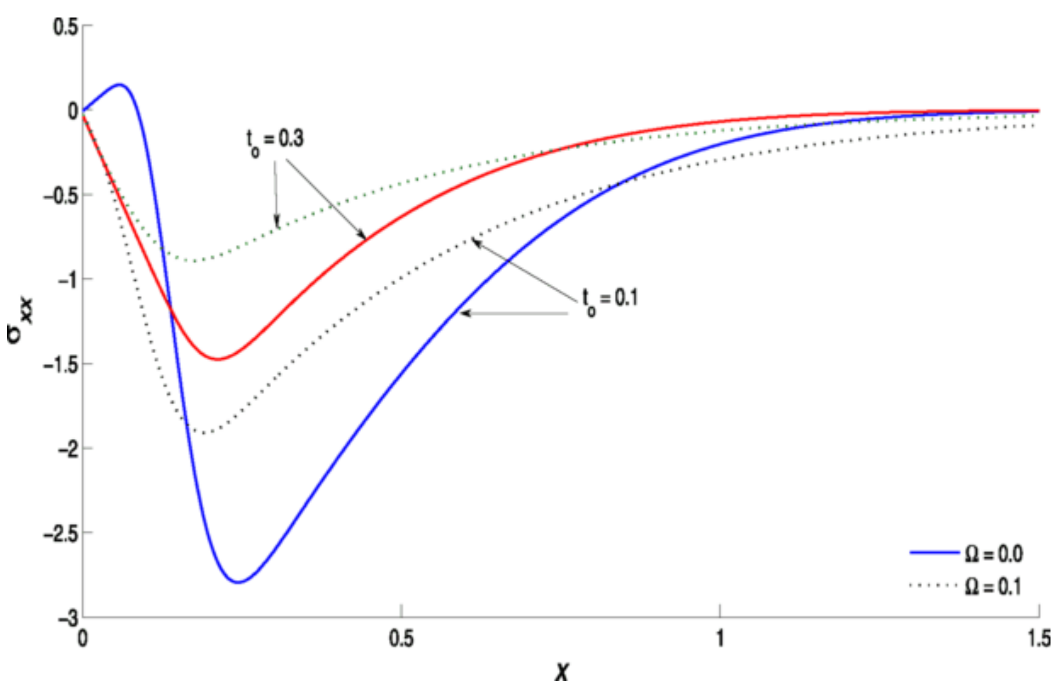


Fig. 5

The stress σ_{xx} distribution for different values of t_o and Ω

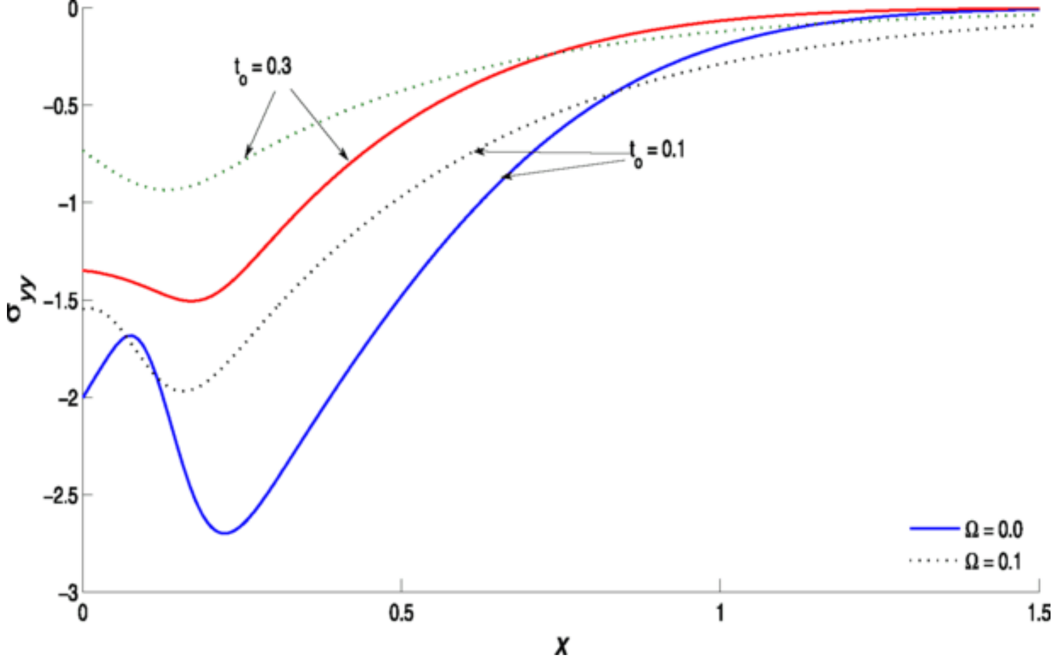


Fig. 6

The stress σ_{yy} distribution for different values of t_o and Ω

7 Conclusion

This paper indicates that, the two-temperature theory of generalized thermoelasticity describes the behavior of the field quantities of a body more realistically than one-temperature theory of generalized thermoelasticity (LS theory). Comparisons with predictions were also made in which there was a two-temperature parameter term, and we have found that this parameter has a significant effect on all the fields and on the speed of the wave propagation. So, according to the results of this work, it is important to distinguish between the dynamical temperature and the conductive temperature.

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